A Study of The Fundamentals of Hypersoft Set Theory

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Abstract- In this study, we discuss the fundamentals of hypersoft set such as hypersoft subset, complement, not hypersoft set and aggregation operators. After that we discuss the hypersoft set relation, sub relation, complement relation, function, matrices and operations on hypersoft matrices.

Keywords-Hypersoft set, hypersoft subset, complement, not set, aggregation operators, hypersoft set relation, sub relation, complement relation, function, hypersoft matrices.

1 Introduction

THE soft-set concept was developed by [1] as a completely new math tool for solving difficulty in dealing with uncertainty. Molodtsov[1] defined a soft set that is sub-set as a parameterized family of the set of the universe where each element is considered a set approximate elements of the soft set. In the past few years, the fundamentals of soft set theory have been studied by different researchers. Magi et al.[2] presented a detailed theoretical study of soft sets, which includes subsets and super set of soft sets, equations of soft sets, operation soft sets such as unions, intersections, and more among others.

He also studied and talked the main features of these operations. Pei and Miao [3] discussed the relationship between soft sets and information system. Ali et al.[4] introduced something new operations such as restricted union, restricted intersection, restricted spacing and extension discusses the intersection of two soft sets and their basics characteristics. A gentle development by Cagman and Enginoglu [5] in matrix theory and successfully applied it to a decision making the problem. Babitha and Sunil [6] discussed the concept of soft-set relation and function. Also, Many related concepts like equivalence soft, soft relationships, soft sets distribution, ordering on soft sets. In the continuation of their work, Babita and Sunil [7] more introduced the work on soft set relation, antisymmetric relation and transitive closure. A soft-set relationship was introduced by Yang and Guo [8]. Sezgin, A. and Atagun [9][, Ge and Yang [10]], Fu Li[11] et al. modified some of Maji's et al. [2] work and also set some new results. Sezgin and Atagun [9] also introduced a restricted symmetric difference of soft sets and examples. Singh and Onyeozili [12] obtained some results with respect to distribution and absorption properties with respect to different operations on soft sets. Singh and Onyeozili [12] also proved that the actions apply on soft sets is similar to actions on soft matrices.Sqlain et. al [20] gave the application of generalized fuzzy topsis in DM for neutrosophic soft set to predict the the champion of FIFA 2018. After that Florentin Samarandache a pioneer of hypersoft set theory and introduced many results about hypersoft sets. He opened many fields in this areas.Saqlain et. al. worked on the generalization of TOPSIS for neutrosophic hyper soft set using accuracy function. [21]. In this paper, we discuss the fundamentals of hypersoft set such as hypersoft subset, complement, not standard aggregation opera-

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tors. After that we discuss the hypersoft set relation, sub relation, complement relation, function, matrices and operations on hypersoft matrices. The rest of this article is structured as follows: Section 2 gives some basic definitions and results on hypersoft sets. Section 3 discusses the various works of relaxation in detail. Section 4 describes many hypersoft set properties without proof set operation. Section 5 focuses on hypersoft set relationships and functions. Final section 6 which consists of two the first sections talk about soft matrix and its basics The second section focuses on work their characteristics.

2 Preliminaries

2.1 Hypersoft set

Let $a_1, a_2, a_3, \cdots, a_n$ be the distinct attributes whose attribute values belongs to the sets $A1, A2, A3, \cdots, An$ respectively, where $A_i \cap A_j = \phi$ for $i \neq j$. A pair $(\Phi, E_1 *`E_2 * E_3 * \cdots * E_n)$ is called a hypersoft set over the universal set U, where Φ is the mapping given by $\Phi : E_1 * E_2 * E_3 * \cdots * E \longrightarrow P(U)$

Example 1 *Let $U = \{R_1, R_2, R_3, R_4, R_5\}$ is universal set, where R_1, R_2, R_3, R_4, R_5 represents the refrigerator. Mr. X, Mrs. X goes to market and wants to buy such refrigerator which is feasible and having more characteristics then that their expectation level. Let a_1= Size, a_2=Pressure, a_3= Freezing point, a_4= Price, be the attributes whose attribute values belongs to the sets B_1, B_2, B_3, B_4 given as B_1= {small= e_1,medium= e_2, large= e_3} B_2 = {Low freezing point = e_4} B_3 = {High expection pressure = e_5, Low condensing pressures = e_6} B_4 = {Low price =e_7} and hypers soft set can be written as*

$$\begin{aligned} & (\Phi, A_1 * A_2 * A_3 * A_4) \\ = & \{\Phi(e_1, e_4, e_5, e_7), \Phi(e_1, e_4, e_6, e_7), \\ & \Phi(e_3, e_4, e_5, e_7), \Phi(e_3, e_4, e_6, e_7)\} \\ = & \{\{R_1, R_2, R_3\}, \{R_1, R_2, R_4\}, \\ & \{R_3, R_5\}, \{R_1, R_2, R_3\}\} \end{aligned}$$

2.2 Hypersoft subset

Assume that $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ and $(\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ be the two hypersoft sets over the same universal sets U. (a) $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ is the hypersoft subset of $(\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ denoted $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n) \subseteq (\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ if (i) $A_1 * A_2 * A_3 * \cdots * A_n \subseteq B_1 * B_2 * B_3 * \cdots * B_n$ and (ii) $\forall\, e \in A_1 * A_2 * A_3 * \cdots * A_n$,$\Phi(e)$ and $\Psi(e)$ are identical approximations. (b) $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ is hypersoft equal set to $(\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ and it is denoted by $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n) = (\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ if $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n) \subseteq (\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ and $(\Psi, B_1 * B_2 * B_3 * \cdots * B_n) \subseteq (\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$.

Example 2 *Let $U = \{R_1, R_2, R_3, R_4, R_5\}$ is universal set, where R_1, R_2, R_3, R_4, R_5 represents the refrigerator. Let a_1= Size, a_2=Pressure, a_3= Freezing point, a_4= Price, be the attributes whose attribute values belongs to the sets B_1, B_2, B_3, B_4 given as B_1= {small= e_1,medium= e_2,large= e_3} B_2 = {Low freezing point = e_4}*
B_3 ={High expectation pressure = e_5, Low condensing pressures = e_6} B_4 = {Low price =e_7} $(\Psi, B_1 * B_2 * B_3 * B_4) = \{\Psi(e_1, e_4, e_5, e_7), \Psi(e_1, e_4, e_6, e_7), \Psi(e_3, e_4, e_5, e_7), \Psi(e_3, e_4, e_6, e_7)\}$
$= \{\{R_1, R_2, R_3\}, \{R_1, R_2, R_4\}, \{R_3, R_3\}, \{R_1, R_2, R_3\}\}$ and $(\Phi, A_1 * A_2 * A_3 * A_4) = \{\Phi(e_1, e_4, e_5, e_7), \Phi(e_1, e_4, e_6, e_7), \Phi(e_3, e_4, e_5, e_7)\}$
$= \{\{R_1, R_2, \}, \{R_2, R_4\}, , \{R_1, R_2, R_3\}\}$ then $(\Phi, A_1 * A_2 * A_3 * A_4) \subset (\Psi, B_1 * B_2 * B_3 * B_4)$, since $A_1 * A_2 * A_3 * A_4 \subseteq B_1 * B_2 * B_3 * B_4$

2.3 Fuzzy hypersoft set

Let $F(U)$ be the set of all fuzzy subsets in the universal set U, let $E_1 * E_2 * E_3 * \cdots * E_n$ be a set of parameters. A pair $(\Phi, E_1 * E_2 * E_3 * \cdots * E_n)$ is called a fuzzy hypersoft set over U, where F is the mapping given by $\Phi : E_1 * E_2 * E_3 * \cdots * E_n \longrightarrow F(U)$ In general, $\Phi(\epsilon) = \{(x, \Phi(\epsilon)(x)/x \in U\}$ here $\epsilon \in E_1 * E_2 * E_3 * \cdots * E_n$. It is very convenient to see that every fuzzy hypersoft set can be seen as an (fuzzy) information system and it can be represented in an data table belonging to the unit interval $[0, 1]$.

Example 3 *Let* $U = \{R_1, R_2, R_3, R_4, R_5\}$ *is universal set , where* R_1, R_2, R_3, R_4, R_5 *represents refrigerator, Let* a_1*= Size,* a_2*=Pressure,* a_3*= Freezing point,* a_4*= Price, be the attributes whose attribute values belongs to the sets* B_1, B_2, B_3, B_4 *given as* $B_1 = \{$*small=* e_1*,medium=* e_2*, large=* $e_3\}$*,* $B_2 = \{$*Low freezing point =* $e_4\}$*,* $B_3 = \{$*High expectation pressure =* e_5*, Low condensing pressures =* $e_6\}$*,* $B_4 = \{$*Low price =*$e_7\}$
$\Phi_{B_1*B_2*B_3*B_4}(e_1, e_4, e_5, e_7) = \{0.5/u_2, 0.9/u_4\}$,
$\Phi_{B_1*B_2*B_3*B_4}(e_1, e_4, e_6, e_7) = \phi$,
$\Phi_{B_1*B_2*B_3*B_4}(e_3, e_4, e_5, e_7) = U, \Phi_{B_1*B_2*B_3*B_4}(e_3, e_4, e_6, e_7) = \{0.2/u_1, 0.3/u_3\}$,
then the fuzzy hypersoft set $\Phi_{B_1*B_2*B_3*B_4}$ *is written by*
$\Phi_{B_1*B_2*B_3*B_4} = \{((e_1, e_4, e_5, e_7) = \{0.5/u_2, 0.9/u_4\}), ((e_1, e_4, e_6, e_7), \phi), ((e_3, e_4, e_5, e_7), U), ((e_3, e_4, e_6, e_7) = \{0.2/u_1, 0.3/u_3\})\}$

Remark 4 *One can easily notice that from the definition of hypersoft fuzzy set is the mapping* $F : E_1 * E_2 * E_3 * \cdots * E_n \longrightarrow P(U)$$ *, which is binary relation between universal set and the set of parameters* $E_1 * E_2 * E_3 * \cdots * E_n$$ *that is* $\forall\ h_j \in U$, $e_i \in E_1 * E_2 * E_3 * \cdots * E_n$$ *and* $F(e_i)(h_i) = F(E_1 * E_2 * E_3 * \cdots * E_n)$$

2.4 Not Hypersoft

Let $A_1 * A_2 * A_3 * \cdots * A_n$ be a set of parameters and not set is denoted by $\rightharpoondown A_1 * A_2 * A_3 * \cdots * A_n$

Example 5 *If*

$$\begin{aligned} & (A_1 * A_2 * A_3 * A_4) \\ = & \{(e_1, e_4, e_5, e_7), (e_1, e_4, e_6, e_7), \\ & (e_2, e_4, e_5, e_7), (e_2, e_4, e_6, e_7), \\ & (e_3, e_4, e_5, e_7), (e_3, e_4, e_6, e_7)\} \end{aligned}$$

and not set can be written as

$$\begin{aligned} \rightharpoondown & (A_1 * A_2 * A_3 * A_4) \\ = & \{\rightharpoondown (e_1, e_4, e_5, e_7), \rightharpoondown (e_1, e_4, e_6, e_7), \\ \rightharpoondown & (e_2, e_4, e_5, e_7), \rightharpoondown (e_2, e_4, e_6, e_7), \\ \rightharpoondown & (e_3, e_4, e_5, e_7), \rightharpoondown (e_3, e_4, e_6, e_7)\} \end{aligned}$$

2.5 Complement of hypersoft set

If $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ be the hypersoft set and complement is denoted by $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)^c$ and it is defined in such a way that $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)^c = (\Phi^c, \rightharpoondown A_1 * A_2 * A_3 * \cdots * A_n)$, where $\Phi : A_1 * A_2 * A_3 * \cdots * A_n \longrightarrow P(U)$ be a mapping as follows $\Phi^c(\alpha) = U - \Phi(\rightharpoondown \alpha)$, $\forall \alpha \in \rightharpoondown A_1 * A_2 * A_3 * \cdots * A_n$

Example 6 *Let* $U = \{R_1, R_2, R_3, R_4, R_5\}$ *is universal set and* $A_1 * A_2 * A_3 * A_4$ *be the set of parameters. Now we defined the hypersoft set on it*

$$\begin{aligned} & (\Phi, A_1 * A_2 * A_3 * A_4) \\ = & \{((e_1, e_4, e_5, e_7), \{R_1, R_2\}), ((e_3, e_4, e_5, e_7), \\ & \{R_1, R_3, R_5\}), ((e_3, e_4, e_6, e_7), \\ & \{R_3, R_4\})\} \end{aligned}$$

then complement of it is

$$\begin{aligned} & (\Phi, A_1 * A_2 * A_3 * A_4)^c \\ = & \{(\rightharpoondown (e_1, e_4, e_5, e_7), \{R_1, R_2\}), \\ (\rightharpoondown & (e_3, e_4, e_5, e_7), \{R_1, R_3, R_5\}), (\rightharpoondown (e_3, e_4, e_6, e_7), \\ & \{R_3, R_4\})\} \end{aligned}$$

2.6 Relative Complement of hypersoft

If $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ be the hypersoft set and relative complement is denoted by $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)^r = (\Phi^r, \rightharpoondown A_1 * A_2 * A_3 * \cdots * A_n)$, where $\Phi : A_1 * A_2 * A_3 * \cdots * A_n \longrightarrow P(U)$ be a mapping as follows $\Phi^r(\alpha) = U - \Phi(\rightharpoondown \alpha)$, $\forall \rightharpoondown \alpha \in A_1 * A_2 * A_3 * \cdots * A_n$

Example 7 *Let* $U = \{R_1, R_2, R_3, R_4, R_5\}$ *is universal set and* $A_1 * A_2 * A_3 * A_4$ *be the set of parameters. Now we defined the hypersoft set on it*

$$\begin{aligned} & (\Phi, A_1 * A_2 * A_3 * A_4) \\ = & \{((e_1, e_4, e_5, e_7), \{R_1; R_2\}), ((e_3, e_4, e_5, e_7) \\ & , \{R_1, R_3, R_5\}), ((e_3, e_4, e_6, e_7), \{R_3, R_4\})\} \end{aligned}$$

then relative complement of it is

$$\begin{aligned} & (\Phi, A_1 * A_2 * A_3 * A_4)^r \\ = & \{((e_1, e_4, e_5, e_7), \{R_3, R_4, R_5\}), \\ & ((e_3, e_4, e_5, e_7), \{R_2, R_4\}), \\ & ((e_3, e_4, e_6, e_7), \{R_1, R_2, R_5\})\} \end{aligned}$$

Proposition 8 *Assume that If* $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ *be the hypersoft set over the universe* U*. Then*

1. $((\Phi, A_1 * A_2 * A_3 * \cdots * A_n)^c)^c = (\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$
2. $((\Phi, A_1 * A_2 * A_3 * \cdots * A_n)^r)^r = (\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$
3. $U^c_{A_1 * A_2 * A_3 * \cdots * A_n} = \Phi^c_{A_1 * A_2 * A_3 * \cdots * A_n} = U^r_{A_1 * A_2 * A_3 * \cdots * A_n}$
4. $\Phi^c_{A_1 * A_2 * A_3 * \cdots * A_n} = U^c_{A_1 * A_2 * A_3 * \cdots * A_n} = \Phi^r_{A_1 * A_2 * A_3 * \cdots * A_n}$

2.7 Aggregation operator of hypersoft sets

Union of hypersoft sets

Assume that $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ and $(\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ be the two hypersoft sets over the same universal sets U, then union between them is denoted by $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n) \cup (\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ is hypersoft set (F, C), where $C = (A_1 * A_2 * A_3 * \cdots * A_n) \cup (B_1 * B_2 * B_3 * \cdots * B_n)$ and $\forall\ e \in C\ F(e) =$

$$\begin{cases} \Phi(e), \text{if } e \in (A_1 * A_2 * A_3 * \cdots * A_n) - \\ \quad (B_1 * B_2 * B_3 * \cdots * B_n) \\ \Psi(e), \text{ if } e \in (B_1 * B_2 * B_3 * \cdots * B_n) - \\ \quad (A_1 * A_2 * A_3 * \cdots * A_n) \\ \Phi(e) \cup \Psi(e), \text{ if } (A_1 * A_2 * A_3 * \cdots * A_n) \cap \\ \quad (B_1 * B_2 * B_3 * \cdots * B_n) \end{cases}$$

Example 9 *Let* $U = \{R_1, R_2, R_3, R_4, R_5\}$ *is universal set and* $A_1 * A_2 * A_3 * A_4$ *and* $B_1 * B_2 * B3 * B_4$ *be the set of parameters. Now we defined the hypersoft sets on it,*

$$\begin{aligned} & (\Phi, A_1 * A_2 * A_3 * A_4) \\ = & \{((e_1, e_4, e_5, e_7), \{R_1; R_2\}), \\ & ((e_3, e_4, e_5, e_7), \{R_1, R_3, R_5\}), \\ & ((e_3, e_4, e_6, e_7), \{R_3, R_4\})\} \end{aligned}$$

and

$$\begin{aligned} & (\Psi, B_1 * B_2 * B_3 * B_4) \\ = & \{((e_1, e_4, e_6, e_7), \{R_4, R_5\}), \\ & ((e_3, e_4, e_5, e_7), \{R_2, R_3\})\} \end{aligned}$$

Then union between them is given as follows

$$\begin{aligned} & (\Phi, A_1 * A_2 * A_3 * A_4) \cup (\Psi, B_1 * B_2 * B_3 * B_4) \\ = & \{((e_1, e_4, e_5, e_7), \{R_1; R_2\}), ((e_3, e_4, e_5, e_7), \\ & \{R_1, R_2, R_3, R_5\}), ((e_3, e_4, e_6, e_7), \\ & \{R_3, R_4\}), ((e_1, e_4, e_6, e_7), \{R_4, R_5\})\} \end{aligned}$$

Intersection of hypersoft sets

Assume that $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ and $(\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ be the two hypersoft sets over the same universal sets U, then intersection between them is denoted by $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n) \cap (\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ is hypersoft set (F, C), where $C = (A_1 * A_2 * A_3 * \cdots * A_n) \cap (B_1 * B_2 * B_3 * \cdots * B_n)$ and $\forall\ e \in C$, $F(e) = \Phi(e) \cap \Psi(e)$

Example 10 *Let* $U = \{R_1, R_2, R_3, R_4, R_5\}$ *is universal set and* $A_1 * A_2 * A_3 * A_4$ *and* $B_1 * B_2 * B3 * B_4$ *be the set of parameters. Now we defined the hypersoft sets on it,*

$$(\Phi, A_1 * A_2 * A_3 * A_4)$$

$$\begin{aligned} = & \{((e_1, e_4, e_5, e_7), \{R_1; R_2\}) \\ & , ((e_3, e_4, e_5, e_7), \{R_1, R_3, R_5\}), \\ & ((e_3, e_4, e_6, e_7), \{R_3, R_4\})\} \end{aligned}$$

and

$$\begin{aligned} & (\Psi, B_1 * B_2 * B_3 * B_4) \\ = & \{((e_1, e_4, e_6, e_7), \{R_4, R_5\}), \\ & ((e_3, e_4, e_5, e_7), \{R_2, R_3\})\} \end{aligned}$$

Then intersection between them is given as follows

$$(\Phi, A_1 * A_2 * A_3 * A_4) \cap (\Psi, B_1 * B_2 * B_3 * B_4) = \{((e_3, e_4, e_5, e_7), \{R_3\})\}$$

2.8 Hypersoft Set Relation and Function

Cartesian Product of hypersoft sets

Assume that $(\Phi, A_1*A_2*A_3*\cdots*A_n)$ and $(\Psi, B_1*B_2*B_3*\cdots*B_n)$ be the two hypersoft sets over the same universal sets U. Then the Cartesian product is denoted by $(\Phi, A_1*A_2*A_3*\cdots*A_n)*(\Psi, B_1*B_2*B_3*\cdots*B_n)$ is hypersoft set $(F, (A_1*A_2*A_3*\cdots*A_n)*(B_1*B_2*B_3*\cdots*B_n))$, where $F: (A_1*A_2*A_3*\cdots*A_n) \cap (B_1*B_2*B_3*\cdots*B_n) \longrightarrow P(U*U)$ and $F(x;y) = \Phi(x)*\Psi(y)$, $\forall\ (x;y) \in (A_1*A_2*A_3*\cdots*A_n)*(B_1*B_2*B_3*\cdots*B_n)$ $F(x;y) = f(k_i, k_j) : k_i \in F(x)$ and $k_j \in F(y)$

Hypersoft Set Relation

$(\Phi, A_1*A_2*A_3*\cdots*A_n)$ and $(\Psi, B_1*B_2*B_3*\cdots*B_n)$ be the two hypersoft sets over the same universal sets U. Then the relation from $(\Phi, A_1*A_2*A_3*\cdots*A_n)$ to $(\Psi, B_1*B_2*B_3*\cdots*B_n)$ is called a hypersoft set relation (R, C) or it is in simple way R is a hypersoft subset and it is denoted by $(\Phi, A_1*A_2*A_3*\cdots*A_n)*(\Psi, B_1*B_2*B_3*\cdots*B_n)$, where $C \subseteq (A_1*A_2*A_3*\cdots*A_n)*(B_1*B_2*B_3*\cdots*B_n)$ and $\forall\ (x,y) \in C$ $R(x,y) = H(x,y)$, where $x = (a_1, a_2, a_3, \cdots, a_n)$ and $y = (b_1, b_2, b_3, \cdots, b_n)$ and $(H, (A_1*A_2*A_3*\cdots*A_n)*(B_1*B_2*B_3*\cdots*B_n)) = (\Phi, A_1*A_2*A_3*\cdots*A_n)*(\Psi, B_1*B_2*B_3*\cdots*B_n)$ A hypersoft set relation on $(\Phi, A_1*A_2*A_3*\cdots*A_n)$ is a hypersoft subset of .$(\Phi, A_1*A_2*A_3*\cdots*A_n)*(\Phi, A_1*A_2*A_3*\cdots*A_n)$. In similar way, the parameterized form of relation R on the hypersoft set $(F, A_1*A_2*A_3*\cdots*A_n)$ is defined as follows. If $(\Phi, A_1*A_2*A_3*\cdots*A_n) = \{\Phi(a), \Phi(b)\}$, then $\Phi(a)R\Phi(b) \Leftrightarrow \Phi(a)*\Phi(b) \in R$

Domain, Range and Inverse of Hypersoft Set

Let R be the hypersoft set relation from $(\Phi, A_1*A_2*A_3*\cdots*A_n)$ to $(\Psi, B_1*B_2*B_3*\cdots*B_n)$ such that $(\Phi, A_1*A_2*A_3*\cdots*A_n)*(\Psi, B_1*B_2*B_3*\cdots*B_n) = (F, (A_1*A_2*A_3*\cdots*A_n)*(B_1*B_2*B_3*\cdots*B_n))$ then (a) The domain of R(dom R) is the hypersoft set $(D, C_1*C_2*C_3*\cdots*C_n) \widetilde{\subset} (\Phi, A_1*A_2*A_3*\cdots*A_n)$ where $C_1*C_2*C_3*\cdots*C_n = F(x,y) \in R$ for some x $\in A_1*A_2*A_3*\cdots*A_n$ and $y \in (\Psi, B_1*B_2*B_3*\cdots*B_n)$ and $D(a) = \Phi(a_1)\ \forall a_1 \in A_1*A_2*A_3*\cdots*A_n$ (b) The range of R(ran R) is the hypersoft set $(D, X_1*X_2*X_3*\cdots*X_n) \widetilde{\subset} B_1*B_2*B_3*\cdots*B_n$ where $X_1*X_2*X_3*\cdots*X_n = H(x,y) \in R$ for some $x \in A_1*A_2*A_3*\cdots*A_n$ and $y \in B_1*B_2*B_3*\cdots*B_n$ and $E(b_1) = G(a_1)\ \forall b_1 \in X_1*X_2*X_3*\cdots*X_n$ (c) The inverse of R denoted by R^{-1} is a hypersoft set relation from $(\Psi, B_1*B_2*B_3*\cdots*B_n)$ to $(\Phi, A_1*A_2*A_3*\cdots*A_n)$ defined by $R^{-1} = \{(\Psi(y), \Phi(x)) : \Phi(x)R\Psi(y)\}$

2.8.1 Sub relation

Let R_1, R_2 be two hypersoft relations on a hypersoft set $(\Phi, A_1*A_2*A_3*\cdots*A_n)$, $R_1 \subset R$, $\forall x, y \in A_1*A_2*A_3*\cdots*A_n$ $\Phi(x)*\Phi(y) \in R_1 \Longrightarrow \Phi(x)*\Phi(y) \in R_2$

Complement of Relation

Let R_1 be a hypersoft relations on a hypersoft set $(\Phi, A_1*A_2*A_3*\cdots*A_n)$, then complement of R_1 is R'_1 is defined as$R'_1 = \{\Phi(x)*\Phi(y) : \Phi(x)*\Phi(y) \notin R_1\}$

Example 11 *Let U denotes the washing machineU* $=\{w_1, w_2, w_3, w_4\}$ *, let* a_1 *= Size,* a_2*= color,* a_3 *= country made , be the attributes whose attribute values belongs to* $A_1 = \{$*Small* $=e_1$*, Medium* $= e_2$*, Large* $= e_3\}$*,* $A_2 = \{$*White* $= e_4$*, Yellow*$= e_5\}$*, and* $A_3 = \{$*Pakistan* $= e_6$*, Japan* $=e_7\}$ *respectively. Then the hypersoft set is given by* $(\Phi, A_1*A_2*A_3) = \{\Phi(e_1, e_4, e_6) = \{w_1, w_2\}, \Phi(e_2, e_4, e_7) = \{w_1, w_4\}, \Phi(e_3, e_5, e_6) = \{w_2, w_3\}\}$ *, let* b_1 *= Services,* b_2 *=vacuum system, be the attributes whose attribute values belongs to* B_1 *={ Self services system=* e'_1*, Water recycling =* $e'_2\}$*, and* B_2 *={ Excellent vacuum system ==* e'_3*, Self service=* $e'_4\}$ *respectively. Then the hypersoft set is given* (Ψ, B_1*B_2)*=*$\{\Phi(e'_1, e'_3) = \{w_1, w_2, w_3\}, \Phi(e'_1, e'_4) = \{w_2, w_3, w_4\}, \Phi(e'_2, e'_3) = \{w_2, w_3\}\}$*, if we want to define a relation R from* $(\Phi, A_1*A_2*A_3)$ *to* (Ψ, B_1*B_2) *in such a way* $\Phi(x)R\Phi(y)$ *if* $\Phi(x) \subseteq \Phi(y)$ *then (a)* $R = \{\Phi(e_1, e_4, e_6)*\Phi(e'_1, e'_3), \Phi(e_3, e_5, e_6)*\Phi(e'_2, e'_3)\}$ *(b) Dom R* $= (D, C_1*C_2*C_3) = \{\{(e_1, e_4, e_6), (e_3, e_5, e_6)\} \subseteq A_1*A_2*A_3\}$ *and* $D(a) = \Phi(a), \forall a \in A_1*A_2*A_3$ *(c) Ran R* $= (E, X1*X_2) = \{\{(e'_1, e'_3), (e'_2, e'_3)\} \subseteq B_1*B_2\}$

and $E(b) = \Psi(b), \forall b \in X1 * X_2$ *(d)*$R' = \{\Psi(e_1', e_3') * \Phi(e_1, e_4, e_6), \Psi(e_2', e_3') * \Phi(e_3, e_5, e_6)\}$

2.9 Hypersoft function

Let $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ and $(\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ be the two hypersoft sets over the same universal sets U. Then the hypersoft relation from $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ to $(\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ defined as $h : (\Phi, A_1 * A_2 * A_3 * \cdots * A_n) \longrightarrow (\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ is called a hypersoft set function. If every element of domain has unique element in range of h. If it is closed $\Phi(x) h \Psi(y)$ i.e $\Phi(x) * \Psi(y) \in h$ for $x \in A_1 * A_2 * A_3 * \cdots * A_n$ and $y \in B_1 * B_2 * B_3 * \cdots * B_n$ then we can represent it in the form $h(\Phi(x)) = .\Psi(y)$

Example 12 *From the previous example the hypersoft function over the universe U is given by*

(i) $h = \{\Phi(e_1).\Psi(e_1'), \Phi(e_2).\Psi(e_1'), \Phi(e_3), \Psi(e_2')\}$

(ii) $h = \{\Phi(e_2).\Psi(e_2'), \Phi(e_3)\Psi(e_1')\}$

but $\{\Phi(e_1).\Psi(e_1'), \Phi(e_1).\Psi(e_2'), \Phi(e_2), \Psi(e_2')\}$ is not function

One-One, Onto, Bijection

A function from $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ to $(\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ is said to be (i) Injective(One to one), $\Phi(x) \neq \Phi(y) \implies h(\Phi(x)) \neq h(\Phi(y))$ (ii) Surjective(On to), If range h $= (\Psi, B_1 * B_2 * B_3 * \cdots * B_n)$ (iii) Bijective, If h is both one-one and onto.

Example 13 *From the previous example (i) is on to but (ii) is not onto*

2.10 Identity hypersoft function

The identity hype soft function I on a hypersoft set $(\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ is defined as $I : (\Phi, A_1 * A_2 * A_3 * \cdots * A_n) \longrightarrow (\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ such that $I(\Phi(a)) = \Phi(a), \forall\, a \in (\Phi, A_1 * A_2 * A_3 * \cdots * A_n)$ where $a \in A_1 * A_2 * A_3 * \cdots * A_n$

2.11 Hypersoft Matrices

Let U be universe of discourse, let $a_1, a_2, a_3, ..., a_n$ be the attributes whose corresponding attribute values belongs the set $E_1, E_2, E_3, ..., E_n$ respectively. Let $A_1 * A_2 * A_3 * \cdots * A_n \subseteq E_1 * E_2 * E_3 * \cdots * E_n$ and $(\Phi_{A_1 * A_2 * A_3 * \cdots * A_n}, E_1 * E_2 * E_3 * \cdots * E_n)$ be the hypersoft set over the universal set U. Then a relation $R_{A_1 * A_2 * A_3 * \cdots * A_n}$ of $U * (E_1 * E_2 * E_3 * \cdots * E_n)$ is defined as below $R_{A_1 * A_2 * A_3 * \cdots * A_n} = \{(u, e) : e \in A_1 * A_2 * A_3 * \cdots * A_n, u \in f_{A_1 * A_2 * A_3 * \cdots * A_n}(e)$. The characteristic function of $R_{A_1 * A_2 * A_3 * \cdots * A_n}$ is defined as, $\zeta_{A_1 * A_2 * A_3 * \cdots * A_n} : U * A_1 * A_2 * A_3 * \cdots * A_n \longrightarrow [0, 1]$

$$H(x) = \begin{cases} 1 & \text{if} \quad (u, e) \in R_{A_1 * A_2 * A_3 * \cdots * A_n} \\ 0 & \text{if} \quad (u, e) \notin R_{A_1 * A_2 * A_3 * \cdots * A_n} \end{cases} \tag{1}$$

Then a hypersoft set $(\Phi_{A_1 * A_2 * A_3 * \cdots * A_n}, E_1 * E_2 * E_3 * \cdots * E_n)$ can be represented unique in the form of matrix and its is denoted by$[x_{ij}]_{m \times n}$

$$[x_{ij}]_{m \times n} = \begin{pmatrix} x11 & x12 & \cdots & x1n \\ x21 & x22 & \cdots & x2n \\ \vdots & \vdots & \vdots & \vdots \\ x, m1 & xm2 & \cdots & xmn \end{pmatrix}$$

where $x_{ij} = \zeta_{A_1 * A_2 * A_3 * \cdots * A_n}(u_i, e_j), e_j \in A_1 * A_2 * A_3 * \cdots * A_n$

Example 14 *From the previous example, if we take $A_1' * A_2' * A_3' \subseteq A_1 * A_2 * A_3$ then the hypersoft set is given as follows $(f_{A_1' * A_2' * A_3'}, A_1 * A_2 * A_3)$ $(\Phi, A_1 * A_2 * A_3) = \{\Phi(e_1, e_4, e_6) = \{w_1, w_2\}, \Phi(e_1, e_4, e_7) = \{w_1, w_4\}, \Phi(e_3, e_4, e_6) = \{w_2, w_3\}, \Phi(e_3, e_4, e_7) = \emptyset\}$ The relation of $(\Phi, A_1 * A_2 * A_3)$ is given by $R_{A_1' * A_2' * A_3'} = \{(e_1, e_4, e_6) = \{w_1\}, (e_1, e_4, e_6) = \{w_2\} = (e_1, e_4, e_7) = \{w_1\}, (e_1, e_4, e_7) = \{w_4\}, \Phi(e_3, e_4, e_6) = \{w_2\}, \Phi(e_3, e_4, e_6) = \{w_3\}\}$ We can write hypersoft matrix as follows $[x_{ij}] =$*

$$\begin{pmatrix} \cdot & (e_1, e_4, e_6) & (e_1, e_4, e_7) & (e_3, e_4, e_6) & (e_3, e_4, e_7) \\ w_1 & 1 & 1 & 0 & 0 \\ w_2 & 1 & 0 & 1 & 0 \\ w_3 & 0 & 0 & 1 & 0 \\ w_4 & 0 & 1 & 0 & 0 \end{pmatrix}$$

2.12 Special hypersoft Matrices

Let the set of all hypersoft matrices over the universal set of discourse U be denoted $HSM(U) m \times n$ or just $(HSM(U)$. Suppose $[x_{ij}] \in HSM(U)$. Then $[x_{ij}]$ is called (a) A zero hypersoft matrix denoted by [0], if $[x_{ij}] = 0\ \forall i$ and j. (b) An A universal hypersoft matrix denoted by $[x_{ij}]$, if $[x_{ij}] = 1$, $\forall j \in I_{A_1*A_2*A_3*\cdots*A_n} = \{J : e_j \in A_1 * A_2 * A_3 * \cdots * A_n$. It is noted that it is occur only for the parameter appearing in the set $A_1 * A_2 * A_3 * \cdots * A_n \subseteq E_1 * E_2 * E_3 * \cdots * E_n$, and it is denoted by $[I]$, if $[x_{ij}] = 1, \forall i$ and j.

Example 15 *From the previous example, let* $[x_{ij}]$, $[y_{ij}]$, $[z_{ij}] \in HSM(U)_{4\times 4}$ *(i)*

$$\zeta_{A_1*A_2*A_3}(e_1, e_4, e_6) = \phi$$
$$\zeta_{A_1*A_2*A_3}(e_1, e_4, e_7) = \phi$$
$$\zeta_{A_1*A_2*A_3}(e_3, e_4, e_6) = \phi$$

, then $[x_{ij}]$ *is the zero hypersoft set matrix given by*

$$[0] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

If $C_1 * C_2 * C_3 \subseteq A'_1 * A'_2 * A'_3$ *as* $C_1 * C_2 * C_3 = \{\{(e_1, e_4, e_6), (e_1, e_4, e_7)\}$, $\zeta_{C_1*C_2*C_3}(e_1, e_4, e_6) = \zeta_{C_1*C_2*C_3}(e_1, e_4, e_7) = U$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

(iii) If $C = A_1 * A_2 * A_3$, $\zeta_{C_1*C_2*C_3}(e_i) = U$ *for each iwhere* $e_i \in A_1 * A_2 * A_3$*, then* $[z_{ij}] = [I]$ *is the universal hypersoft matrix given by*

$$I = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

2.13 Hypersoft Sub Matrices

Let $P = [x_{ij}]$,$Q = [y_{ij}] \in HSM(U)$. Then (i) We call P is hypersoft matrix of Q, denoted by$P \subseteq Q$ if $x_{ij} \leq y_{ij}$ for each $i$and j. In this case, we also say that P is dominated by Q. On the other hand one can say that Q dominates P. Now we define P and Q are comparable and it is denoted by P//Q iff $P \subseteq Q$ or $Q \subseteq P$ (ii) P is proper hypersoft sub matrix of Q if $[x_{ij}] \subset [y_{ij}]$ and for at least one term $x_{ij} < y_{ij}$ for all i and j. In this case, we call P is properly dominated by Q. (iii) P is strictly proper hypersoft sub matrix of Q and it is denoted by $P \subsetneq Q$, if $P \subset Q$ and for each i and j. In this case, we call P is strictly dominated by Q. (iv) If P and Q are said to be hypersoft equal matrices if $x_{ij} = y_{ij}$ and it is denoted by $P \cong Q$ for each i and $j$and it is equivalent to $P \widetilde{\subseteq} Q$ and $Q \widetilde{\subseteq} P$, then $P \cong Q$ It is prompted to observe that$\widetilde{\subseteq}$ is a partial ordering (reflexive, Anti symmetric, and transitive) on the class of hypersoft matrices.

2.14 Operations on Hypersoft Matrices

Now we are going to discuss the concept of union, intersection, complement, difference and product of hypersoft matrices and their fundamental features. Let $P = [x_{ij}]$, $Q = [y_{ij}] \in HSM(U)$ is called the (i) Union of P and Q denoted by $P \cup Q$ if $z_{ij} = \max \{x_{ij}, y_{ij}\}$ for all i and j. (ii) And intersection of two hypersoft set is denoted by $P \cap Q$, where $z_{ij} = \min \{x_{ij}, y_{ij}\}$ for all i and j and it is called disjoint if $P \cap Q = \phi$ (iii) Complement of P is denoted by P^c, if $z_{ij} = 1 - x_{ij}$ for all i and j. (iv) Difference b/w P and Q is denoted by P/Q or $P - Q$ which is also called relative complement of Q in P.

Example 16 *Let*

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

and

$$Q = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

then

(i)

$$P \cup Q = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(ii)

$$P \cap Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

(iii)

$$P^c = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

(iv)

$$Q^c = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

(v)

$$P - Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

(vi)

$$Q - P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Characteristics of hypersoft Matrix Operations

Let $P = [x_{ij}], Q = [y_{ij}]$ and $R = [z_{ij}] \in HSM(U)$ (i) $P \widetilde{\cup} P = P$,$P \widetilde{\cap} P = P$(Idempotent law) (ii) $P \widetilde{\cup} \phi = P$, $P \widetilde{\cap} U = P$ (Identity law)

2.14.1 Product of hypersoft set

Let $P = [x_{ij}], Q = [y_{ij}]$ and $R = [z_{ik}] \in HSM(U)_{m\times n}$ (i) And product is the function that maps element from $HSM(U)_{m\times n} \times HSM(U)_{m\times n}$ to the set $HSM(U)_{(m\times n)^2}$ that in such a way that $[x_{ij}] \bigwedge [y_{ik}] = [z_{ir}]$, where $z_{ir} = \min\{x_{ij}, y_{ik}\}$ and $r = n(j\text{-}1) + k$ (ii) OR product of P and Q is the function between $HSM(U)_{m\times n} \times HSM(U)_{m\times n}$ to the set $HSM(U)_{m\times n}$ matrix and it is denoted by $P \bigvee Q$. It can be represented in such a way that $[x_{ij}] \bigvee [y_{ik}] = [z_{ir}]$, where $z_{ir} = \max\{x_{ij}, y_{ij}\}$ and $r = n(j\text{-}1) + k$ (iii) And Not product of P and Q is the function between $HSM(U)_{m\times n} \times HSM(U)_{m\times n}$ to the set $HSM(U)_{(m\times n)^2}$ matrix and it is denoted by $P \overline{\bigwedge} Q$. It can be represented in such a way that $[x_{ij}] \overline{\bigwedge} [y_{ik}] = [z_{ir}]$,where $z_{ir} = \min\{x_{ij}, 1 - y_{ij}\}$ and $r = n(j\text{-}1) + k$ (iv) OR Not product of P and Q, denoted by $P \overline{\bigvee} Q$ is defined by $HSM(U)_{m\times n} \times HSM(U)_{m\times n} \longrightarrow HSM(U)_{m\times n}$ in such a way that $[x_{ij}] \overline{\bigvee} [y_{ik}] = [z_{ir}]$ where $z_{ir} = \max\{x_{ij}, 1 - y_{ij}\}$ and $r = n(j\text{-}1) + k$

Remark 17 *If the two matrices have same order then the product hold.*

2.15 Features of Hypersoft Matrices

Let $P = [x_{ij}], Q = [y_{ij}] \in HSM(U)_{m\times n}$. Then the following features hold
i $(P \bigvee Q)^c = P^c \bigwedge Q^c$, $(P \bigwedge Q)^c = P^c \bigvee Q^c$(De Morgan law)
ii

$(P \overline{\bigvee} Q)^c = P^c \overline{\bigwedge} Q^c$, $(P \overline{\bigwedge} Q)^c = P^c \overline{\bigvee} Q^c$(De Morgan law) This can be proved easily by using definition.

2.15.1 Example

Suppose $P = [x_{ij}], Q = [y_{ij}] \in HSM(U)_{3\times 3}$, given by $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

$P \bigwedge Q = [x_{ij}] \bigwedge [y_{ik}] = [z_{ir}]_{3\times 9} = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$P^c = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and $Q^c = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

$(P \bigwedge Q)^c = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

and $P^c \bigvee Q^c = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$ which shows that De Morgan laws are valid for product of hypersoft matrix.

3 Conclusions

In this paper, we have discussed the fundamentals of hypersoft set such as hypersoft subset, complement, not set and aggregation operators. After that we have discussed the hypersoft set relation, sub relation, complement relation, function, matrices and operations on hypersoft matrices. In this study, we also observed that some properties of classical sets do not hold for hypersoft set operations. This results will be very fruitful for future experts to enhance the work for fuzzy hypersoft set, intuitionistic fuzzy hypersoft set, neutrosophic fuzzy hypersoft set, Plithogenic hypersoft set and hypersoft multi sets among others.

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